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# PLANE STRAIN DEFORMATION IN AN ORTHOTROPIC MICROPOLAR THERMOELASTIC SOLID WITH A HEAT SOURCE

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*The problem of plane strain deformation in an orthotropic micropolar generalized thermoelastic half-space subjected to an arbitrary point heat source is solved. Closed-form solutions for spatial distributions of displacements, stresses, and temperature are derived by using the Fourier transform. A numerical inversion technique has been applied to obtain the solution in the physical domain. Numerical results are obtained and presented graphically along with a comparison of the ones for concentrated and distributed, as well as mechanical and thermal, sources.*

***Keywords:*** *Orthotropic micropolar thermoelastic solid; point heat source.*

**Introduction.** The theory of micropolar elasticity introduced and developed by Eringen [1] aroused great interest because of its possible utility in investigating the deformation properties of solids for which the classical theory is inadequate. The micropolar theory is believed to be particularly useful in investigating materials consisting of bar-like molecules which exhibit microrotation effects and can transmit body and surface couples.

The earth is generally assumed in both theory and practical application to be isotropic or, at the least, composed of isotropic layers. Sufficiently detailed studies, however, are often indicative of the presence of anisotropy. This situation usually manifests itself in the directional dependence of seismic body waves, but there are other anomalous observations which indirectly suggest a departure from isotropy.

Iesan [2–4] analyzed the static problem of plane micropolar strain of a homogeneous and orthotropic elastic solid, the torsion problem of homogeneous and orthotropic cylinders in the linear theory of micropolar elasticity, and bending of orthotropic micropolar elastic beams by terminal couples. Nakamura et al. [5] derived the finite element method for orthotropic micropolar elasticity. Recently Kumar et al. [6–8] discussed various problems in an orthotropic micropolar continuum.

The coupling between the strain and temperature fields was first studied by Duhamel [9], who derived the equations for the distribution of strains in an elastic medium subjected to temperature gradient. Biot [10] justified and derived, on the basis of irreversible thermodynamics, the fundamental equations of thermoelasticity and stated its variation principles. For static problems this coupling vanishes and the thermal field becomes independent of the strain field. Several papers have been published by various authors with account for the coupling (the works of Nowacki [11], Dhaliwal and Singh [12]).

The linear theory of micropolar thermoelasticity developed by Eringen [13] and Nowacki [14] to include thermal effects is known as micropolar coupled thermoelasticity. A comprehensive work has been done in micropolar thermoelasticity theory. Recently Kumar and Ailawalia [15–17] studied various problems in a micropolar thermoelastic material possessing cubic symmetry.

The influence of a point heat source is an important topic for studying the failure mechanism of high temperature composites. This is mainly due to the fact that the contribution of point heat sources to material failure can be pronouncedly influenced by various homogeneities in the material. Generally, the interaction arises from two sources: the misfit in the atomic arrangements at the interfaces and mismatch in elastic properties. In view of this, in the present paper we consider the problem of an orthotropic micropolar thermoelastic half-space with an arbitrarily located point heat source. The Fourier transform technique is used to solve it. The transformed components of normal stress, tangential stress, tangential couple stress, and temperature distributions due to mechanical and thermal sources

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are obtained. Such mechanical/thermal loading may produce deformation and temperature rise in a thin zone near the contact zone. It is therefore useful to analyze this class of problems by using a formulation that is as exact as possible and to provide results for surface and/or near the surface field quantities that may be required for design purposes.

**Basic Equations.** Following Iesan [2] and Dhaliwal and Singh [12], the basic equations of plane strain for a homogeneous orthotropic micropolar thermoelastic solid in the absence of body forces and body couples can be written as

$$t_{ji,j} = 0 \,, \tag{1}$$

$$m_{i3,i} + \varepsilon_{ij3} t_{ij} = 0 \,, \quad i, j = 1, 2 \,, \tag{2}$$

where the comma in the subscript denotes partial differentiation with respect to a spatial coordinate and $\varepsilon_{ijk}$ is the Levi-Civita permutation tensor defined as

$$\varepsilon_{ijk} = \begin{cases} 1 \,, & \text{if } (i, j, k) \text{ are in even permutation} \,, \\ -1 \,, & \text{if } (i, j, k) \text{ are in odd permutation} \,, \\ 0 \,, & \text{if any two indices have the same value} \,. \end{cases}$$

The heat conduction equation looks like

$$K_1^* \frac{\partial^2 T}{\partial x^2} + K_2^* \frac{\partial^2 T}{\partial y^2} = Q \,. \tag{3}$$

The quantities used are given as follows:

$$t_{11} = A_{11} e_{11} + A_{12} e_{22} - \beta_1 T \,, \quad t_{12} = A_{77} e_{12} + A_{78} e_{21} \,, \quad m_{13} = B_{66} \phi_{3,1} \,, \tag{4}$$

$$t_{22} = A_{12} e_{11} + A_{22} e_{22} - \beta_2 T \,, \quad t_{21} = A_{78} e_{12} + A_{88} e_{21} \,, \quad m_{23} = B_{44} \phi_{3,2} \,, \tag{5}$$

where

$$e_{ij} = u_{j,i} + \varepsilon_{ji3} \phi_3 \,.$$

Here, the relations between $\beta_i$ and the coefficient of thermal expansion $\alpha_i$ are

$$\beta_1 = A_{11} \alpha_1 + A_{12} \alpha_2 \,, \quad \beta_2 = A_{21} \alpha_1 + A_{22} \alpha_2 \,.$$

**Formulation and Solution of the Problem.** A homogeneous orthotropic micropolar generalized thermoelastic medium in a rectangular Cartesian coordinate system ($x$, $y$, $z$) with $y$ axis directed vertically into the medium is considered when the latter is initially in an undisturbed state and at uniform temperature $T_0$. Since we are considering the two-dimensional plane strain parallel to the $xy$ plane, we assume that the components of the displacement vector $\mathbf{u}$ and microrotation vector $\phi$ are of the form

$$\mathbf{u} = (u, v, 0) \,, \quad \phi = (0, 0, \phi_z) \,. \tag{6}$$

The distributions of the displacement components $u$ and $v$, microrotation component $\phi_z$ and temperature $T$ depend on $x$, $y$, and $t$ and are independent of the coordinate $z$, so that $\frac{\partial}{\partial z} \equiv 0$. With these considerations and using (4)—(6) in Eqs. (1) and (2), we obtain the equations of the plane strain in the form

$$\left( A_{11} \frac{\partial^2}{\partial x^2} + A_{88} \frac{\partial^2}{\partial y^2} \right) u + (A_{12} + A_{78}) \frac{\partial^2 v}{\partial x \partial y} - K_1 \frac{\partial \phi_z}{\partial y} - \beta_1 \frac{\partial T}{\partial x} = 0 \,,$$

$$(A_{12}+A_{78})\frac{\partial^2 u}{\partial x\partial y}+\left(A_{77}\frac{\partial^2}{\partial x^2}+A_{22}\frac{\partial^2}{\partial y^2}\right)v-K_2\frac{\partial\phi_z}{\partial x}-\beta_2\frac{\partial T}{\partial y}=0\ ,$$

$$K_1\frac{\partial u}{\partial y}+K_2\frac{\partial v}{\partial x}+\left(B_{66}\frac{\partial^2}{\partial x^2}+B_{44}\frac{\partial^2 u_2}{\partial y^2}-\chi\right)\phi_z=0\ . \tag{7}$$

Here the coefficients used enter into the expressions

$$t_{yx}=A_{78}\left(\frac{\partial v}{\partial x}-\phi_z\right)+A_{88}\left(\frac{\partial u}{\partial y}+\phi_z\right),$$

$$t_{yy}=A_{12}\frac{\partial u}{\partial x}+A_{22}\left(\frac{\partial u}{\partial y}+\phi_z\right), \tag{8}$$

$$m_{yz}=B_{44}\frac{\partial\phi_z}{\partial y}\ ,$$

whereas

$$K_1=A_{78}-A_{88}\ ,\ \ K_2=A_{77}-A_{78}\ ,\ \ \chi=K_2-K_1\ .$$

Further we consider a point heat source of constant strength $Q_0$ acting at the origin of the coordinates, so that

$$Q=Q_0\delta\,(x)\ , \tag{9}$$

where $\delta(x)$ is the Dirac delta function.

We define the dimensionless variables as

$$(x',y')=\frac{(x,y)}{l}\ ,\ \ (u',v')=\frac{(u,v)}{l}\ ,\ \ \phi_z'=\frac{B_{44}}{K_1l^2}\phi_z\ ,\ \ t_{ij}'=\frac{t_{ij}}{A_{22}}\ ,\ \ m_{yz}'=\frac{lm_{yz}}{B_{44}}\ ,$$

$$T'=\frac{T}{T_0}\ ,\ \ Q'=\frac{Q}{Q_0}\ ,\ \ \overline{K}=\frac{K_1^*}{K_2^*}\ ,\ \ \overline{\beta}=\frac{\beta_2}{\beta_1}\ , \tag{10}$$

where $l$ is a parameter with the dimensionality of length. With the help of Eqs. (9) and (10), Eqs. (3) and (7) are reduced to the form (with dropping of the primes)

$$\left(\frac{\partial^2}{\partial y^2}+\frac{A_{11}}{A_{88}}\frac{\partial^2}{\partial x^2}\right)u+\frac{A_{12}+A_{78}}{A_{88}}\frac{\partial^2 v}{\partial x\partial y}-\frac{K_1^2l^2}{B_{44}A_{88}}\frac{\partial\phi_z}{\partial y}-\frac{\beta_1T_0}{A_{88}}\frac{\partial T}{\partial x}=0\ , \tag{11}$$

$$\frac{A_{12}+A_{78}}{A_{22}}\frac{\partial^2 u}{\partial x\partial y}+\left(\frac{A_{77}}{A_{22}}\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)v-\frac{K_1K_2l^2}{A_{22}B_{44}}\frac{\partial\phi_z}{\partial x}-\frac{\beta_2T_0}{A_{22}}\frac{\partial T}{\partial y}=0\ , \tag{12}$$

$$\frac{K_1l^2}{B_{44}}\frac{\partial u}{\partial y}+\frac{K_2l^2}{B_{44}}\frac{\partial v}{\partial x}+\frac{K_1l^2}{B_{44}}\left(\frac{B_{66}}{B_{44}}\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}-\frac{\chi l^2}{B_{44}}\right)\phi_z=0\ , \tag{13}$$

$$\frac{1}{\overline{K}}\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=\frac{Q_0 l^2}{T_0 K_2^*}\,Q\,. \tag{14}$$

Applying the Fourier transform defined as

$$\tilde{f}(\xi, x_2, t)=\int_{-\infty}^{\infty}\overline{f}(x_1, x_2, t)\exp(i\xi x_1)\,dx_1 \tag{15}$$

to Eqs. (11)—(14), we obtain

$$\left(\frac{d^2}{dy^2}-\xi^2 d_1 d_4\right)\tilde{u}-i\xi\,(d_2+d_3)\frac{d\tilde{v}}{dy}-d_6\,(d_3-1)\frac{d\tilde{\phi}_z}{dy}+d_8 i\xi\tilde{T}=0\,, \tag{16}$$

$$-\frac{i\xi\,(d_2+d_3)}{d_4}\frac{d\tilde{u}}{dy}+\left(\frac{d^2}{dy^2}-\frac{\xi^2 d_5}{d_4}\right)\tilde{v}-\frac{i\xi d_6\,(d_5-d_3)}{d_4}\tilde{\phi}_z-\frac{\overline{\beta} d_8}{d_4}\frac{dT}{dy}=0\,, \tag{17}$$

$$\frac{d\tilde{u}}{dy}-\frac{i\xi d_7\tilde{v}}{d_6}+\left(\frac{d^2}{dy^2}-\xi^2 d_9-d_{10}\,(d_5-2d_3+1)\right)\tilde{\phi}_z=0\,, \tag{18}$$

$$\left(\frac{d^2}{dy^2}-\frac{\xi^2}{\overline{K}}\right)\tilde{T}=a_1\tilde{Q}\,, \tag{19}$$

where

$$d_1=\frac{A_{11}}{A_{22}},\ \ d_2=\frac{A_{12}}{A_{88}},\ \ d_3=\frac{A_{78}}{A_{88}},\ \ d_4=\frac{A_{22}}{A_{88}},\ \ d_5=\frac{A_{77}}{A_{88}},\ \ d_6=\frac{K_1 l^2}{B_{44}},\ \ d_7=\frac{K_2 l^2}{B_{44}}\,,$$

$$d_8=\frac{\beta_1 T_0}{A_{88}},\ \ d_9=\frac{B_{66}}{B_{44}},\ \ d_{10}=\frac{A_{88}}{B_{44}},\ \ a_1=\frac{Q_0 l^2}{K_1^*\xi^2 T_0}\,.$$

The roots of Eq. (19) are equal to $\pm\xi/\sqrt{K}$. Let us assume that the solution of Eq. (19) satisfying the condition according to which $\tilde{T}\to 0$ as $y\to\infty$ is

$$\tilde{T}=A_4\exp\left(\frac{-\xi y}{\sqrt{\overline{K}}}\right)-a_1\tilde{Q}\,, \tag{20}$$

where $A_4$ is an arbitrary constant to be determined from the boundary conditions. Substituting (20) into Eqs. (16)–(18), we obtain

$$\left(\frac{d^2}{dy^2}-\xi^2 d_1 d_4\right)\tilde{u}-i\xi\,(d_2+d_3)\frac{d\tilde{v}}{dy}-d_6\,(d_3-1)\frac{d\tilde{\phi}_z}{dy}=-A_4 d_8 i\xi\exp\left(\frac{-\xi y}{\sqrt{\overline{K}}}\right)+\frac{d_8 i\xi Q_0 l^2}{K_1^*\xi^2 T_0}\tilde{Q}\,,$$

$$-\frac{i\xi\,(d_2+d_3)}{d_4}\frac{d\tilde{u}}{dy}+\left(\frac{d^2}{dy^2}-\frac{\xi^2 d_5}{d_4}\right)\tilde{v}+\frac{d_6\,(d_5-d_3)\,i\xi}{d_4}\tilde{\phi}_z=-\frac{A_4\overline{\beta} d_8\xi}{d_4\sqrt{\overline{K}}}\exp\left(\frac{-\xi y}{\sqrt{\overline{K}}}\right), \tag{21}$$

$$\frac{d\tilde{u}}{dy} + \left(\frac{d^2}{dy^2} - \frac{i\xi d_7}{d_6}\right)\tilde{v} - \left(\xi^2 d_9 - l^2 d_{10}(d_5 - 2d_3 + 1)\right)\tilde{\phi}_z = 0 .$$

The system of equations (21) have nontrivial solution if and only if the determinant of the coefficients at $\tilde{u}, \tilde{v}, \tilde{\phi}_z$ vanishes, which yields the equation

$$\left(\frac{d^6}{dy^6} + A\frac{d^4}{dy^4} + B\frac{d^2}{dy^2} + C\right)(\tilde{u}, \tilde{v}, \tilde{\phi}_z) = \left(-A_4 d_8 i\xi \exp\left(\frac{-\xi y}{\sqrt{K}}\right) + a_5, A_4 a_6 \exp\left(\frac{-\xi y}{\sqrt{K}}\right), 0\right), \tag{22}$$

where

$$A = -\xi^2 d_9 - l^2 d_{10}(d_5 - 2d_3 + 1) - \xi^2\left(\frac{d_5}{d_4} + d_1 d_4 - \frac{(d_2 + d_3)^2}{d_4} + d_6(d_3 - 1)\right),$$

$$B = \frac{\xi^2}{d_4}\Big\{\xi^2 d_5 d_9 + l^2 d_{10}(d_5 - 2d_3 + 1)\left(d_1 d_4^2 - (d_2 + d_3)^2 + d_5\right) + (d_5 - d_3)(d_7 + d_6(d_2 + d_3))$$

$$+ \xi^2\left(d_1 d_4^2 d_9 + d_1 d_4 d_5 - d_9(d_2 + d_3)^2\right) + \frac{d_7 d_6 (d_2 + d_3)(d_3 - 1)}{d_6} - d_6 d_5 (d_3 - 1)\Big\},$$

$$C = \xi^2 d_1\left(d_5 d_9 + l^2 d_5 d_{10}(d_5 - 2d_3 + 1) - d_7(d_5 - d_3)\right),$$

$$a_5 = \frac{i d_8 Q_0 \tilde{Q}}{K_1^* \xi T_0}, \quad a_6 = -\frac{d_8 \xi \bar{\beta}}{d_4\sqrt{K}} .$$

Let us assume that $q_i$ ($i$ = 1, 2, 3) are the roots of Eq. (22). The solution of this equation satisfying the condition $\tilde{u}, \tilde{v}, \tilde{\phi}_z \to 0$ as $y \to \infty$ is

$$\tilde{u} = \sum_{i=1}^{3} A_i s_i \exp(-q_i y) + a_7 - A_4 a_8 \exp\left(\frac{-\xi y}{\sqrt{K}}\right),$$

$$\tilde{v} = \sum_{i=1}^{3} A_i r_i \exp(-q_i y) - A_4 a_9 \exp\left(\frac{-\xi y}{\sqrt{K}}\right),$$

$$\tilde{\phi}_z = \sum_{i=1}^{3} A_i \exp(-q_i y), \tag{23}$$

where $A_i$ are arbitrary constants to be determined by using the boundary conditions,

$$a_7 = \frac{d_8 i l^2 Q_0 \tilde{Q}}{K_1^* \xi T_0 q_1^2 q_2^2 q_3^2}, \quad a_8 = \frac{d_8 i \xi}{\prod_{i=1}^{3}\left(\frac{\xi^2}{K} - q_i^2\right)}, \quad a_9 = \frac{d_8 \xi \bar{\beta}}{d_4\sqrt{K}\prod_{i=1}^{3}\left(\frac{\xi^2}{K} - q_i^2\right)},$$

$$s_i = \frac{-i\xi (d_2 + d_3)\, r_i q_i - d_6 (d_3 - 1)\, q_i}{q_i^2 - \xi^2 d_1 d_4},$$

$$r_i = \frac{i\xi d_6 \left((d_2 + d_3)\left(q_i^2 - \xi^2 d_9 - l^2 d_{10} (d_5 + 2d_3 - 1)\right) + d_6 (d_5 - d_3)\right)}{\xi^2 (d_6 d_5 - d_7 (d_2 + d_3)) - q_i^2 d_4 d_6}. \tag{24}$$

**The Case of Mechanical Force.** Here the boundary conditions on the surface $y = 0$ are

$$t_{yy} = -P\psi(x), \quad t_{yx} = 0, \quad m_{yz} = 0, \quad T = 0, \tag{25}$$

where $\psi(x)$ specifies the vertical traction distribution function along the $x$ axis and $P$ is the magnitude of the applied mechanical force. Applying the Fourier transform defined by Eq. (15) to the boundary conditions (25) and using (8), (10), and (23), we obtain the expressions for the normal stress, tangential stress, tangential couple stress, and temperature distributions for an orthotropic micropolar thermoelastic solid as

$$\tilde{t}_{yy} = \frac{1}{\Delta}\left(\Delta_1 c_1^* \exp(-q_1 y) + \Delta_2 c_2^* \exp(-q_2 y) + \Delta_3 c_3^* \exp(-q_3 y)\right) + \Delta_4 a_{10} \exp\left(\frac{-\xi y}{\sqrt{\bar{K}}}\right) + a_{11}\tilde{Q}, \tag{26}$$

$$\tilde{t}_{yx} = \frac{1}{\Delta}\left(\Delta_1 a_1^* \exp(-q_1 y) + \Delta_2 a_2^* \exp(-q_2 y) + \Delta_3 a_3^* \exp(-q_3 y)\right) + \Delta_4 a_{12} \exp\left(\frac{-\xi y}{\sqrt{\bar{K}}}\right), \tag{27}$$

$$\tilde{m}_{yz} = \frac{1}{\Delta}\left(\Delta_1 b_1^* \exp(-q_1 y) + \Delta_2 b_2^* \exp(-q_2 y) + \Delta_3 b_3^* \exp(-q_3 y)\right), \tag{28}$$

$$\tilde{T} = \Delta_4 \exp\left(\frac{-\xi y}{\sqrt{\bar{K}}}\right) - a_1 \tilde{Q}, \tag{29}$$

where

$$a_i^* = \frac{-d_3 (i\xi r_i + d_6) - q_i s_i + d_6}{d_4}, \quad b_i^* = -d_6 q_i, \quad c_i^* = -\frac{i\xi d_2 s_i + q_i r_i d_4}{d_4}, \quad i = 1, 2, 3,$$

$$a_2 = \Delta_4 a_{10} + a_{11}\tilde{Q}, \quad a_3 = \Delta_4 a_{12}, \quad a_{12} = \frac{-i\xi^2 d_8 \left(1 + \frac{\bar{\beta} d_3}{d_4^2}\right)}{\sqrt{\bar{K} \prod_{i=1}^{3} \left(\frac{\xi^2}{\bar{K}} - q_i^2\right)}},$$

$$a_{10} = \frac{-\bar{\beta} d_8}{d_4} - \frac{d_8 \xi^2 \left(d_2 + \frac{\bar{\beta}}{\bar{K}}\right)}{d_4 \prod_{i=1}^{3} \left(\frac{\xi^2}{\bar{K}} - q_i^2\right)}, \quad a_{11} = \frac{-\bar{\beta} d_8 a_1}{d_4} + \frac{d_2 d_8 l^2 Q_0}{K_1^* d_4 T_0 q_1^2 q_2^2 q_3^2},$$

$$\Delta = c_1^* (a_2^* b_3^* - a_3^* b_2^*) - c_2^* (a_1^* b_3^* - a_3^* b_1^*) + c_3^* (a_1^* b_2^* - a_2^* b_1^*) ,$$

$$\Delta_1 = (-\tilde{P}\tilde{\psi} + a_2) (a_2^* b_3^* - a_3^* b_2^*) - c_2^* a_3^* b_1^* + c_3^* a_3^* b_2^* ,$$

$$\Delta_2 = c_1^* a_3^* b_3^* - c_2^* (a_1^* b_3^* - a_3^* b_1^*) + c_3^* (a_1^* b_2^* - a_2^* b_1^*) ,$$

$$\Delta_3 = a_3^* (c_1^* b_2^* + c_2^* b_1^*) + (-\tilde{P}_1 \tilde{\psi} + a_2) (a_1^* b_2^* - b_1^* a_2^*) , \quad \Delta_4 = a_1 \tilde{Q} .$$

*The concentrated force* is described by substituting the Dirac delta function

$$\psi(x) = \delta(x) ,$$

where

$$\tilde{\psi}(\xi) = 1 , \tag{30}$$

into Eq. (25).

For *the uniformly distributed force* which is applied to a strip of nondimensional width $2a$, we have

$$\psi(x) = H(x + a) - H(x - a) . \tag{31}$$

Using the Fourier transform in Eq. (31) we obtain

$$\tilde{\psi}(\xi) = 2 \sin(\xi a)/\xi . \tag{32}$$

The solution due to *the linearly distributed force* applied to the half-space is obtained by substituting

$$\psi(x) = \begin{cases} 1 - \dfrac{|x|}{a} , & \text{if } |x| \le a , \\ 0 , & \text{if } |x| > a . \end{cases} \tag{33}$$

Applying the Fourier transform in the case of the strip of nondimensional width $2a$ at the origin of the coordinate system $x = y = 0$ to Eq. (33), we have

$$\tilde{\psi}(\xi) = \frac{2 [1 - \cos(\xi a)]}{\xi^2 a} . \tag{34}$$

The expressions for stress and temperature distributions can be obtained for concentrated, uniformly distributed, and linearly distributed forces by substituting $\tilde{\psi}(\xi)$ from (30), (32) and (34), respectively, into Eqs. (26)—(29). For numerical computations, we have taken the value of $\tilde{P}_1$ equal to unity.

**The Case of Thermal Source.** Here the boundary conditions on the surface $y = 0$ are

$$t_{yy} = 0 , \quad t_{yx} = 0 , \quad m_{yz} = 0 , \quad T = P_2 \eta(x) , \tag{35}$$

where $\eta(x)$ is the known function defined afterwards and $P_2$ is the constant temperature at the boundary surface. For the function $\eta(x)$ we can write the expressions for the *concentrated source, uniformly distributed source*, and *linearly distributed source* analogous to the above expressions for the respective mechanical forces. Then we also apply the Fourier transform to the boundary conditions (35) and obtain the expressions for the stress and temperature distributions in the case of an orthotropic micropolar thermoelastic solid from Eqs. (26)—(29), with $\Delta_4$ replaced by $\Delta_4'$, where

$$\Delta_4' = \tilde{\eta}(x) \tilde{P}_2 + a_1 \tilde{Q} .$$

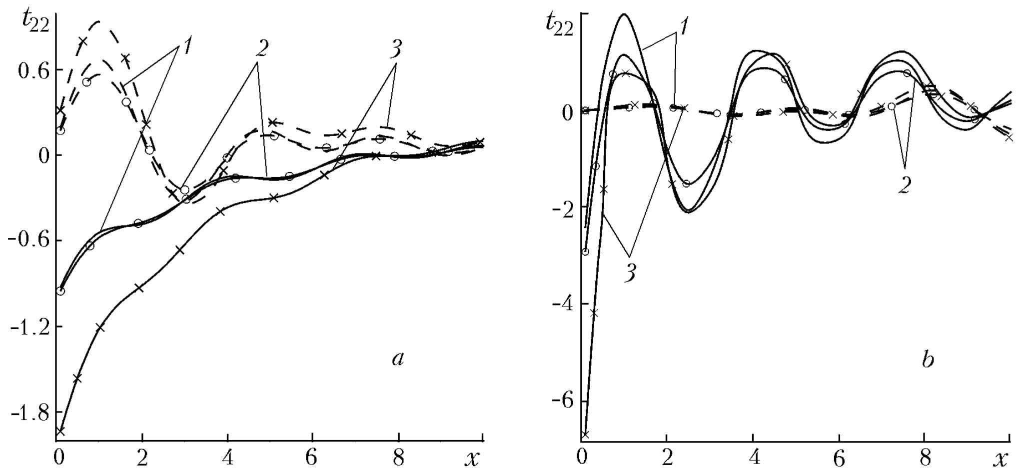


Fig. 1. Normal stress vs. the distance due to a mechanical force (a) and a thermal source (b) for orthotropic (solid curves) and isotropic (dashed curves) solids and various forces and sources: 1) concentrated; 2) linearly distributed; 3) uniformly distributed.

**Inversion of the Transform.** To obtain the solution of the problem in the physical domain, we must invert the transform in Eqs. (26)–(29). These expressions are functions of y and the parameter of the Fourier transform $\xi$, hence are of the form $\tilde{f}(\xi, y)$. To get the function $f(x, y)$ in the physical domain, we invert the Fourier transform using the relations

$$f(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{f}(\xi, y) \exp(-i\xi x)\, d\xi ,$$

$$f(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[ f_e \cos(\xi x) - i \sin(\xi x) f_o \right] d\xi ,$$

where $f_e$ and $f_o$ are the even and odd parts of the function $\tilde{f}(\xi, y)$, respectively. The method for evaluating this integral is described by Press et al. [18] and involves the use of Rhomberg's integration with adaptive size. This also uses the results of successive refinement of the extended trapezoidal rule followed by extrapolation of the results to the limit, when the step size tends to zero.

**Results and Discussion.** For numerical computations we take the following values for ortho-tropic micropolar thermoelastic solid: $A_{11} = 13.97 \cdot 10^9$ N/m$^2$, $A_{22} = 13.75 \cdot 10^9$ N/m$^2$, $A_{12} = 8.13 \cdot 10^9$ N/m$^2$, $A_{77} = 3.0 \cdot 10^9$ N/m$^2$, $A_{88} = 3.2 \cdot 10^9$ N/m$^2$, $A_{78} = 2.2 \cdot 10^9$ N/m$^2$, $B_{44} = 0.056 \cdot 10^5$ N, $B_{66} = 0.057 \cdot 10^5$ N. Confining our study to the isotropic case, we take

$$A_{11} = A_{22} = \lambda + 2\mu + K, \quad A_{77} = A_{88} = \mu + K, \quad A_{12} = \lambda, \quad A_{78} = \mu, \quad -K_1 = K_2 = K = \frac{\chi}{2}$$

and, following Gauthier [19], use the values for aluminum epoxy-like composite as $\lambda = 7.59 \cdot 10^9$ N/m$^2$, $\mu = 1.89 \cdot 10^9$ N/m$^2$, $K = 0.0149 \cdot 10^9$ N/m$^2$, $K^* = 204$ $J$/(sec·m·$^o$C). The results for orthotropic micropolar thermoelastic solid (OMTS) and isotropic micropolar thermoelastic solid (IMTS) are shown in Figs. 1—3. The computations were carried out at $y = 0.1$ within the range $0 \le x \le 10$ for $T = 298$ K.

Figure 1a shows the variation of the normal stress $t_{22}$ with $x$ for mechanical force. For OMTS, the value of normal stress increases monotonically with $x$. Here the values of $t_{22}$ are smaller in the case of uniformly distributed force as compared to two other forces. However, for IMTS, the values of $t_{22}$ increase within the range $0 \le x \le 1$, de-

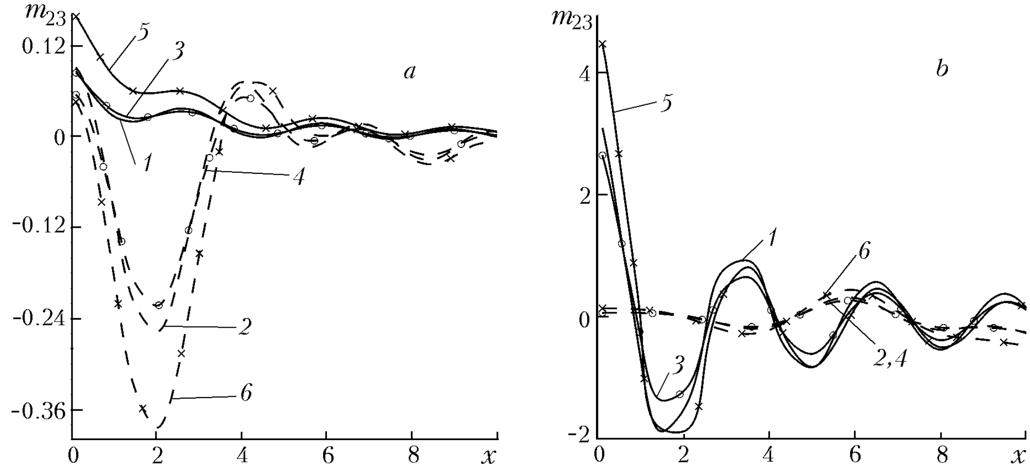


Fig. 2. Tangential couple stress vs. the distance due to a mechanical force (a) and a thermal source (b). The designations are the same as in Fig. 1.

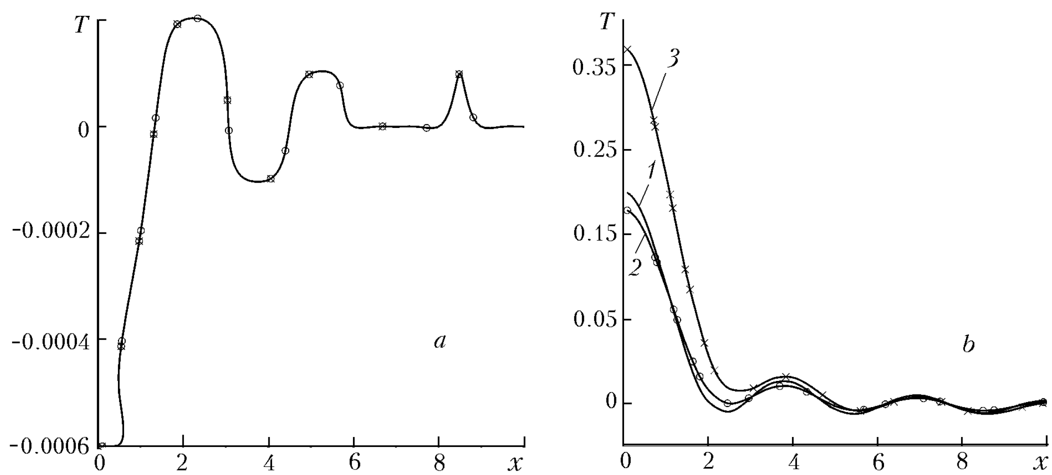


Fig. 3. Temperature vs. the distance due to a mechanical force (a) and a thermal source (b). The designations are the same as in Fig. 1.

crease within the range $1 \le x \le 3$, and then oscillate with increasing $x$ for all kinds of forces applied. Here the value of $t_{22}$ is the largest for a uniformly distributed force.

Figure 1b presents a variation of the normal stress $t_{22}$ with $x$ for thermal sources. For OMTS the values of $t_{22}$ increase sharply within the range $0 \le x \le 1.2$, decrease within the range $1.2 \le x \le 3$, and then oscillate with $x$ for all the sources applied. Here the value of $t_{22}$ is the smallest for a uniformly distributed source. For IMTS, the values of normal stress oscillate with a small amplitude and practically coincide for all the three sources.

It is observed from Fig. 2a that for OMTS the values of the tangential couple stress $m_{23}$ for a mechanical force decrease slowly with $x$. The values of $m_{23}$ are greater in the case of a uniformly distributed force. The values of the tangential couple stress in the case of IMTS decrease within the range $0 \le x \le 2$, increase within the range $2 \le x \le 4$, and then oscillate.

It is evident from Fig. 2b that for all the thermal sources in the case of an orthotropic solid the values of the tangential couple stress $m_{23}$ decrease within the range $0 \le x \le 1.5$, increase within the range $1.5 \le x \le 3.5$, and then oscillate, whereas for an isotropic solid they oscillate everywhere over the entire range.

Figure 3a shows the temperature distribution $T$ for mechanical forces and indicates that this distribution is independent of anisotropy and the force kind. The temperature distributions for thermal sources are also not affected by anisotropy of the material but are different for various sources (Fig. 3b).

**Conclusions.** The Fourier transform technique was used to derive the expressions for stress and temperature distributions due to mechanical and thermal sources. A significant anisotropy effect has been observed for the normal stress t22 and tangential couple stress m23. At the point of application of the source, the value of the normal stress increases and then oscillates with decreasing amplitude, whereas the reverse behavior is observed for the tangential couple stress. It is observed that the trends of variation of the normal stress t22, tangential couple stress m23, and temperature T distributions are similar with different amplitudes, when mechanical and thermal sources are applied.

## NOTATION

$a$, half-width of a strip; $A_{11}$, $A_{12}$, $A_{22}$, $A_{77}$, $A_{78}$, $A_{88}$, characteristic constants of the material; $B_{44}$, $B_{66}$, material constants; $e_{ij}$, components of the micropolar strain tensor; $K_1^*, K_2^*$, thermal conductivities; $m_{ij}$, components of the tangential couple stress; $P$, mechanical force; $Q$, strength of the applied heat source; $t$, time; $t_{ij}$, components of the normal stress tensor; $T$, temperature; $\mathbf{u}$, displacement vector; $x$, $y$, $z$, coordinates; $\alpha_1$, $\alpha_2$, coefficients of thermal expansion; $\beta_1, \beta_2$, material constants which are associated with the mechanical and thermal properties of the material; $\phi_3$, component of microrotation vector along z axis. Subscripts: e, even; o, odd.

## REFERENCES


1. A. C. Eringen, Linear theory of micropolar elasticity, *J. Math. Mech.*, **15**, 909–923 (1966).
2. D. Iesan, The plane micropolar strain of orthotropic elastic solids, *Arch. Mech.*, **25**, 547–561 (1973).
3. D. Iesan, Torsion of anisotropic elastic cylinders, *ZAMM*, **54**, 773–779 (1974).
4. D. Iesan, Bending of orthotropic micropolar elastic beams by terminal couples, *An. St. Univ. Iasi*, **XX**, 411–418 (1974).
5. S. Nakamuka, R. Benedict, and R. Lakes, Finite element method for orthotropic micropolar elasticity, *Int. J. Eng. Sci.*, **22**, 319–330 (1984).
6. R. Kumar and P. Ailawalia, Effects of fluid layer at micropolar orthotropic boundary surface, in: *Sadhana*, Part 6, **29**, 605–616 (2004).
7. R. Kumar and S. Choudhary, Dynamical behavior of orthotropic micropolar elastic medium, *J. Vibration Control*, **8**, 1053–1069 (2004).
8. R. Kumar and P. Ailawalia, Moving inclined load at boundary surface, *Appl. Math. Eng*., **26**, 476–485 (2005).
9. J. M. C. Duhamel, *J. Ec. Polyt. Paris,* **15**, 1–57 (1837).
10. M. A. Biot, Thermoelasticity and irreversible thermodynamics, *J. Appl. Phys.*, **27**, 240–253 (1956).
11. W. Nowacki, *Dynamic Problems of Thermoelasticity*, Noordhoff Int. Pub., Leyden, Netherlands (1970).
12. R. S. Dhaliwal and A. Singh, *Dynamic Coupled Thermoelasticity*, Hindustan Publication Corporation, New Delhi, India (1980), p. 726.
13. A. C. Eringen, *Foundations of Micropolar Thermoelasticity*, Course of lectures, No. 23, CSIM Udine Springer (1970).
14. M. Nowacki, Couple-stresses in the theory of thermoelasticity, in: *Proc. IUTAM Symposia*, Vienna (1966), pp. 259–278.
15. R. Kumar and P. Ailawalia, Interaction due to mechanical /thermal sources in a micropolar thermoelastic medium possessing cubic symmetry, *IJSS*, **43**, 2761–2798 (2006).
16. R. Kumar and P. Ailawalia, Deformation due to time harmonic sources in micropolar thermoelastic medium possessing cubic symmetry with two relaxation times, *Appl. Math. Mech.*, **27**, No. 6, 781–792 (2006).
17. R. Kumar and P. Ailawalia, Time harmonic sources at micropolar thermoelastic medium possessing cubic symmetry with one relaxation time, *Eur. J. Mech. A/Solids*, **25**, 271–282 (2006).
18. W. H. Press, S. A. Teukolshy, W. T. Vellering, and B. P. Flannery, *Numerical Recipes*, Cambridge University Press, Cambridge (1986).
19. R. D. Gauthier, Experimental investigations on micropolar media, in: O. Brulin and R. K. T. Hsieh (Eds.), *Mechanics of Micropolar Media*, World Scientific, Singapore (1982).